

Impact of a light stabilized radion in supernovae cooling

Prasanta Kumar Das,^{1,*} J. R. Selvaganapathy,¹ Chandradew

Sharma,¹ Tarun Kumar Jha,¹ and V. Sunil Kumar¹

¹*Birla Institute of Technology and Science-Pilani,*

K. K. Birla Goa campus, NH-17B, Zuarinagar, Goa-403726, India

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Abstract

In the Randall-Sundrum model where the Standard Model fields are confined to the TeV brane located at the orbifold point $\theta = \pi$ and the gravity peaks at the Planck brane located at $\theta = 0$, the stabilized modulus (radion) field is required to stabilize the size of the fifth spatial dimension. It can be produced copiously inside the supernova core due to nucleon-nucleon bremsstrahlung, electron-positron and plasmon-plasmon annihilations, which then subsequently decays to neutrino-antineutrino pair and take away the energy released in SN1987A explosion. Assuming that the supernovae cooling rate $\dot{\epsilon} \leq 7.288 \times 10^{-27} \text{ GeV}$, we find the lower bound on the radion vev $\langle \phi \rangle \sim 9.0 \text{ TeV}$, 2.2 TeV and 0.9 TeV corresponding to the radion mass $m_\phi = 5 \text{ GeV}$, 20 GeV and 50 GeV, respectively.

Keywords: Randall-Sundrum model, Radion, Supernovae cooling.

*Author(corresponding): pdas@goa.bits-pilani.ac.in

I. INTRODUCTION

Several new models based on extra spatial dimensions have been put forward to explain the large hierarchy between the Plack scale $M_{Pl}(\sim 10^{19}$ GeV) and the electro-weak scale $M_{EW}(\sim 100$ GeV) [1]. Among them the Randall-Sundrum(RS) model is particularly interesting [2] since it solves the hierarchy problem in an elegant manner. According to this model the world is 5-dimensional and the fifth spatial dimension is characterized by the angular coordinate $-\pi \leq \theta \leq \pi$. The space is an S^1/Z_2 orbifold (i.e. the point (x, θ) is identified with the point $(x, -\theta)$). The metric describing such a 5-dimensional world is non-factorizable and a line element in this space-time is given by

$$ds^2 = e^{-2kR_c|\theta|}\eta_{\mu\nu}dx^\mu dx^\nu - R_c^2 d\theta^2 \quad (1)$$

where k is the bulk curvature constant and x^μ are the Lorentz coordinates of four dimensional surfaces of constant θ . This theory postulates two D_3 branes along x^μ directions living in 5-dimensional world: one is located at the orbifold point $\theta = 0$ where gravity peaks(strong) and the other at the orbifold point $\theta = \pi$ where the Standard Model(SM) fields reside and gravity is weak. The factor $e^{-2kR_c|\theta|}$ appearing in the metric is known as the warp factor. The compactification radius $R_c(\sim$ the distance between the two D_3 branes) can be related to the vacuum expectation value (VEV) of the modulus field $T(x)$ which corresponds to the fluctuations of the metric over the background geometry given by R_c . Replacing R_c by $T(x)$, we can rewrite the RS metric at the orbifold point $\theta = \pi$ as

$$ds^2 = g_{\mu\nu}^{vis} dx^\mu dx^\nu - T(x)^2 d\theta^2 \quad (2)$$

where $g_{\mu\nu}^{vis} = e^{-2\pi k T(x)} \eta_{\mu\nu} = \left(\frac{\Phi(x)}{f}\right)^2 \eta_{\mu\nu}$. Here $f^2 = \frac{24M_5^3}{K}$ and M_5 is the 5-dimensional Planck scale. One is thus left with a scalar field $\phi(x)$ which is dubbed as the radion field [3]. The existence of the radion (modulus) field is a direct and straightforward consequence of the existence of the non-factorizable metric. Randall and Sundrum [2] showed that if the above metric Eq.1 be a solution of the 5-dimensional Einstein equations, then k is to be related to the bulk cosmological constant and the vacuum energies of the two D_3 branes in a particular way [2]. The modulus field in the original RS model was massless and it had no potential: so it was not stabilized. One needs to generate a stable vacuum for $T(x)$ at R_c , which in turn can give $\phi(x)$ a non-zero vev. This is done in the Goldberger-Wise

mechanism [3], using a bulk scalar field with suitable interactions with the two 3 branes, whereupon a potential for the modulus field is generated, and one ends up with a radion of nonzero mass. In particular, the radion can be lighter than the other low-lying gravitonic degrees of freedom and can very well act as the first messenger of a scenario with compact extra dimensions, and reveal itself in collider experiments. Several studies on the observable implications of the radion are available in the literature [4].

Beside the collider signals, the model predicts a variety of novel signals which can be tested in a class of astrophysical or cosmological observations. In particular, the energy loss mechanism of the core-collapse SN1987A explosion and its relevance in new physics context is an exciting area to work. A lot of studies in this direction have already been made and are available in the literature [5]. It is interesting to see whether the light stabilized brane-world radion do have some role in the supernovae cooling or not. The present work is intended to explore this possibility.

II. SUPERNOVA EXPLOSION AND COOLING

Supernovae, the final state of an exploding star, come in two main observational varieties: Type II are those whose optical spectra exhibit Hydrogen lines and have less sharp peaks at maxima (of 1 billion solar luminosities), whereas the optical spectra for the Type I supernovae does not have any Hydrogen lines and it exhibits sharp maxima [6]. Physically, there are two fundamental types of supernovae, based on what mechanism powers them: the thermonuclear supernovae and the core-collapse ones. Only supernovae Ia are thermonuclear type and the rest are formed by core-collapse of a massive star. The core-collapse supernovae are the class of explosions which mark the evolutionary end of massive stars ($M \geq 8 M_\odot$). The kinetic energy of the explosion carries about 1% of the liberated gravitational binding energy of about 3×10^{53} ergs and the remaining 99% going into neutrinos. This powerful and detectable neutrino burst is the main astro-particle interest of the core-collapse supernovae.

In the case of SN1987A, about 10^{53} ergs of gravitational binding energy was released in few seconds and the neutrino fluxes were measured by Kamiokande [7] and IMB [8] collaborations. Numerical neutrino light curves can be compared with the SN1987A data where the measured energies are found to be “too low”. For example, the numerical simulation in [9] yields time-integrated values $\langle E_{\nu_e} \rangle \approx 13$ MeV, $\langle E_{\bar{\nu}_e} \rangle \approx 16$ MeV, and $\langle E_{\nu_x} \rangle \approx 23$ MeV. On

the other hand, the data imply $\langle E_{\bar{\nu}_e} \rangle = 7.5$ MeV at Kamiokande and 11.1 MeV at IMB [10]. Even the 95% confidence range for Kamiokande implies $\langle E_{\bar{\nu}_e} \rangle < 12$ MeV. Flavor oscillations would increase the expected energies and thus enhance the discrepancy [10]. It has remained unclear if these and other anomalies of the SN1987A neutrino signal should be blamed on small-number statistics, or point to a serious problem with the SN models or the detectors, or is there a new physics happening in supernovae?

Since we have these measurements already at our disposal, now if we propose some novel channel through which the core of the supernova can lose energy, the luminosity in this channel should be low enough to preserve the agreement of neutrino observations with theory. That is $\mathcal{L}_{new\ channel} \leq 10^{53} \text{ ergs s}^{-1}$. This idea was earlier used to put the strongest experimental upper bounds on the axion mass [11]. In large extra dimension scenario (where the weakness of $4-d$ gravity is obtained by the large size of the extra spatial dimensions via $M_{Pl}^2 = (2\pi R)^d M_D^{d+2}$ [1]), the KK gravitons interact with the strength of ordinary gravitons and thus are not trapped in the supernovae core. During the first few seconds after collapse, the core contains neutrons, protons, electrons, neutrinos and thermal photons(plasmons). There are a number of processes in which higher-dimensional gravitons can be produced. For the conditions that pertain in the core at this time (temperature $T \sim 30 - 70$ MeV, density $\rho \sim (3 - 10) \times 10^{14} \text{ g cm}^{-3}$), the relevant processes are shown below

- (i) Graviton(\mathcal{G}) production in Nucleon-Nucleon Brehmstrahlung: $N + N \rightarrow N + N + \mathcal{G}$
- (ii) Graviton production in photon fusion: $\gamma + \gamma \rightarrow \mathcal{G}$
- (iii) Graviton production in electron-positron annihilation process: $e^- + e^+ \rightarrow \mathcal{G}$
- (iv) Graviton production in plasmon-plasmon(photon inside plasma becomes massive and called plasmon) annihilation process: $\gamma_P + \gamma_P \rightarrow \mathcal{G}$

The constraint on luminosity of this process can be converted into a bound on the $4+d$ dimensional Planck scale M_D . Raffelt has proposed a simple analytic criterion based on detailed supernova simulations [5]: if any energy-loss mechanism has an emissivity greater than $10^{19} \text{ ergs g}^{-1} \text{ s}^{-1}$ then it will remove sufficient energy from the explosion to invalidate the current understanding of Type-II supernovae's neutrino signal.

The dominant process relevant for the SN1987A where the temperature is comparable to m_π and so the strong interaction between N's is unsuppressed. This process can be represented as (see above)

$$N + N \rightarrow N + N + \mathcal{G}, \tag{3}$$

where N can be a neutron or a proton and \mathcal{G} is a higher-dimensional graviton.

Table 1

Group/Collaboration	M_D (GeV)	d
Cullen <i>et al.</i> [12]	≥ 50 TeV, ≥ 4 TeV, ≥ 1 TeV	2, 3, 4
Barger <i>et al.</i> [13]	≥ 51 TeV, ≥ 3.6 TeV	2, 3
Hannestad <i>et al.</i> [14]	≥ 84 TeV, ≥ 7 TeV	2, 3

Table 1: The lower bound on the higher dimensional Planck scale M_D corresponding to the no. of extra spatial dimensions d is shown. The lower bound follows from the fact for any new physics channel contributing to the SN1987A energy loss, the loss rate $\dot{\epsilon} \leq 7.288 \times 10^{-27} \text{ GeV}$.

For the core temperature $T = 30$ MeV and $\rho = 3 \times 10^{14} \text{ g cm}^{-3}$, we list in Table 1 the results(lower bound on M_D) of various authors. In addition it is worthwhile to mention that the KK gravitons produced in plasmon-plasmon collision which contributes in the supernovae cooling gives rise strong bound on M_D : for $d = 2$ one finds $M_D \geq 22.5$ TeV and for $d = 3$ one finds $M_D \geq 1.4$ TeV [15].

Another extra-dimensional model that can play a crucial role in this SN1987A cooling is the Randall-Sundrum(RS) model and was first looked at in [16]. The authors in [16] studied the impact of a light radion on neutrino-anti-neutrino oscillation. They found that for a light radion of mass $m_\phi \geq 1$ GeV with $\langle \phi \rangle = 1$ TeV, the interaction potential(arising due to the exchange of a radion between the supernovae matter and the neutrino-antineutrino pair) does not affect the neutrino oscillation. However, the role of a light radion in the Supernovae cooling was not looked at. The present work is an effort in that direction.

We will see that how the radion produced in electron-positron, plasmon-plasmon annihilation takes part in the supernovae SN1987A cooling.

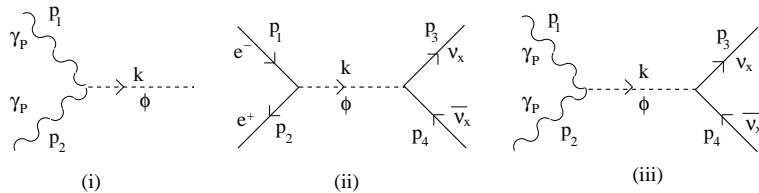


FIG. 1: Feynman diagrams for the processes $\gamma_P + \gamma_P \rightarrow \phi$, $e^- e^+ \xrightarrow{\phi} \nu_x \bar{\nu}_x$ and $\gamma_P + \gamma_P \rightarrow \phi \rightarrow \nu_x + \bar{\nu}_x$ (where $x = \mu, \tau$) which contribute to the SN1987A cooling.

The three primary new physics diagrams that may contribute to the supernovae cooling are:

(i) $\gamma_P + \gamma_P \rightarrow \phi$, (ii) $e^+ + e^- \rightarrow \phi \rightarrow \nu_x + \bar{\nu}_x$ and (iii) $\gamma_P + \gamma_P \rightarrow \phi \rightarrow \nu_x + \bar{\nu}_x$ where $x = \mu, \tau$. These are shown in Fig. 1.

III. METHODOLOGY OF CALCULATION

In Goldberger-Wise mechanism the modulus field $T(x)$ gains a mass and the radion field Φ (where $\Phi = f e^{-\pi k T(x)}$ with $f = M_{Pl}$) couples to the brane matter through the trace of the energy-momentum tensor [3]

$$\mathcal{L} = \frac{\phi}{\langle \phi \rangle} T_\mu^\mu \quad (4)$$

where $\phi = \Phi - \langle \phi \rangle$ is the fluctuation of the radion field from the VEV $\langle \phi \rangle$. In the RS scenario, $\langle \phi \rangle \sim \text{TeV}$ for $k R_c \simeq 12$ in order to produce the weak scale from the Planck scale through the exponential warp factor. Since radion coupling to SM matter is determined by the $4 - d$ general covariance, it's coupling to matter is universal i.e. it couples to the trace T_μ^μ of the energy-momentum tensor $T_{\mu\nu}$ of the SM matter fields which resides on the TeV brane and is given by

$$T_\mu^\mu(SM) = \sum_\psi \left[\frac{3i}{2} (\bar{\psi} \gamma_\mu \partial_\nu \psi - \partial_\nu \bar{\psi} \gamma_\mu \psi) \eta^{\mu\nu} - 4m_\psi \bar{\psi} \psi \right] - 2m_W^2 W_\mu^+ W^{-\mu} - m_Z^2 Z_\mu Z^\mu + (2m_h^2 h^2 - \partial_\mu h \partial^\mu h) + \dots \quad (5)$$

Inside the supernovae, the relevant matter fields are the nucleons, electrons-positrons, plasmons and the interaction of braneworld radion (ϕ) with them is given in Eq. 4.

Now for a generic $2 \rightarrow N$ body scattering, the scattering cross section is given by

$$\sigma = \frac{1}{Flux} \int \prod_f \frac{d^3 p_f}{(2\pi)^3 2E_f} (2\pi)^4 \delta^4 \left(p_1 + p_2 - \sum_f p_f \right) \overline{|\mathcal{M}_{fi}|^2} \quad (6)$$

where $Flux = 4E_1 E_2 v_{rel}$. Here E_1, E_2 are the energies of the incoming particles 1 and 2 whose masses are m_1 and m_2 , respectively and v_{rel} is the relative velocity between them. Defining the energy loss per unit mass $\dot{\epsilon} = \frac{Q}{\rho_{SN}} (\text{erg } g^{-1} s^{-1})$ for a generic $2 \rightarrow N$ scattering contributing to the cooling process at temperature T one finds

$$\dot{\epsilon} = \frac{1}{\rho_{SN}} \prod_{i=1}^2 \int \frac{d^3 \mathbf{p}_i}{2E_i (2\pi)^3} f_i(E_i) \prod_{j=3}^N \int \frac{d^3 \mathbf{p}_j}{2E_j (2\pi)^3} (1 \pm f_j(E_j)) (2\pi)^4 \delta^4 \left(\sum_{i=1}^2 p_i - \sum_{j=3}^N p_j \right) \frac{1}{4} \sum_{spins} |M|^2 \quad (7)$$

where f_i , the occupation numbers for the initial colliding particles (i.e. electrons, positrons, plasmons or nucleons) and $(1 \pm f_j)$, the Pauli blocking factors for the final state bosons (+ sign) and fermions(- sign)

For a general reaction of the kind $a + b \rightarrow c$, the above expression takes the form

$$\sigma = \frac{1}{Flux} |\overline{\mathcal{M}_{fi}}|^2 2\pi \delta(S - m_c^2). \quad (8)$$

In the center of mass(c.o.m) frame, we use the notation \sqrt{S} for the total initial energy

$$\sqrt{S} = E_1 + E_2 \quad (9)$$

$$Flux = 4E_1 E_2 v_{rel} = 4|\mathbf{p}| \sqrt{S}, \quad (10)$$

where $|\mathbf{p}| = |\mathbf{p}_1| = |\mathbf{p}_2| = \frac{\lambda^{1/2}(S, m_1^2, m_2^2)}{2\sqrt{S}}$ and E_1 and E_2 are the energies of the particles 1 and 2. The function $\lambda(x, y, z) (= x^2 + y^2 + z^2 - 2xy - 2yz - 2zx)$, is the standard Källen function.

Now we are concerned with the energy loss to radion. It is convenient and standard [5, 24] to define the quantities $\dot{\epsilon}_{a+b \rightarrow c}$ which is the rate at which energy is lost to radion via the process $a + b \rightarrow c$ where c has a decay width. In terms of the cross-section $\sigma_{a+b \rightarrow c}$ the number densities $n_{a,b}$ for a,b and the mass density ρ , $\dot{\epsilon}$ is given by

$$\dot{\epsilon}_{a+b \rightarrow c} = \frac{\langle n_a n_b \sigma_{(a+b \rightarrow c)} v_{rel} E_{cm} \rangle}{\rho} \quad (11)$$

where the brackets indicate the thermal averaging and $E_{cm}(= E_a + E_b)$ is the center-of-mass(c.o.m) energy of the two colliding particles a and b . Note that in the present case, the final state radion has a smaller decay width but is stable over the size of the neutron star because of its large life time $\sim 10^9 (100 \text{ MeV}/m)^3 \text{ yr}$ (See [23]) and thus it can escape the supernovae while allowing it to cool.

Next we are interested in the processes in which electron-positron and plasmon-plasmon collisions produce radion followed by its decay to muon and tau neutrino anti-neutrino pair. These $2 \rightarrow 2$ (i.e. $a + b \rightarrow c \rightarrow d + g$) scatterings will also contribute in the overall SN1987A cooling. The volume energy-loss rate ($\dot{\epsilon}$) per unit mass of the supernovae matter can be calculated as

$$\dot{\epsilon} = \frac{1}{\rho_{SN}} \prod_{i=1}^2 \int \frac{d^3 \mathbf{p}_i}{2E_i (2\pi)^3} f_i(E_i) \prod_{j=3}^4 \int \frac{d^3 \mathbf{p}_j}{2E_j (2\pi)^3} (1 \pm f_j(E_j)) (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \frac{1}{4} \sum_{spins} |M|^2 \quad (12)$$

where $f_{1,2}$, the occupation numbers for the initial colliding electrons, positrons, plasmons or nucleons and $(1 \pm f_{3,4})$, the Pauli blocking factors of the final state bosons (+ sign) and fermions(− sign).

IV. RADION PRODUCTION IN PLASMON FUSION

Photons are quite abundant in supernovae. Due to plasma effect inside the supernovae, photons becomes effectively massive. These massive photons are called plasmons. Our interest is in the production of a light stabilized radion ϕ in plasmon-plasmon annihilation

$$\gamma_P(p_1) + \gamma_P(p_2) \rightarrow \phi(k). \quad (13)$$

The interaction vertex of the plasmon-plasmon-radion $\gamma_P(p_1) - \gamma_P(p_2) - \phi(k)$ is given by [3]

$$- \frac{2im_A^2}{\langle \phi \rangle} \eta^{\mu\nu} \quad (14)$$

In the c.o.m frame, the 4-momentum vectors associated with the incoming and outgoing particles are given by

$$p_1^\mu = (E_1, 0, 0, p), \quad p_2^\mu = (E_2, 0, 0, -p), \quad (15)$$

$$k^\mu = (E_\phi, 0, 0, 0). \quad (16)$$

where $p = |\vec{p}|$. It often turns out to be more convenient to keep the polarizations explicitly. Here ϵ_μ^\pm and ϵ_μ^0 are the transverse and longitudinal polarization vectors of a massive gauge boson. For a massive vector boson(*e.g.* plasmon) with momentum $k^\mu = (E, 0, 0, k)$ and mass m_A , the components of the polarization vector can be written as

$$\epsilon_\mu^+(k) = \frac{1}{\sqrt{2}}(0, 1, i, 0), \quad (17)$$

$$\epsilon_\mu^-(k) = \frac{1}{\sqrt{2}}(0, -1, i, 0), \quad (18)$$

$$\epsilon_\mu^0(k) = \frac{1}{m_A}(k, 0, 0, -E). \quad (19)$$

The polarization vectors satisfy the following normalization and polarization sum conditions

$$e^{s\mu} e_\mu^{s'*} = 4\delta^{ss'}, \quad \sum_{s=1}^3 e_\mu^s(k) e_\nu^{s*}(k) = -\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m_A^2}, \quad e^{s\mu\nu} e_{\mu\nu}^{s'*} = 4\delta^{ss'}. \quad (20)$$

The total squared amplitude, averaged over the initial three polarizations (massive plasmons have three state of polarizations) states and summed over final states for the process $\gamma_P(k_1) + \gamma_P(k_2) \rightarrow \phi(p)$ is

$$\overline{|M|^2} = \left(\frac{1}{3}\right)^2 \left(\frac{2\omega_0^2}{\langle\phi\rangle}\right)^2 \left\{ 2 + \left(1 - \frac{s}{2\omega_0^2}\right)^2 \right\} \quad (21)$$

Substituting this in Eq. 8, we find the total cross-section $\sigma_{\gamma_P\gamma_P \rightarrow \phi}$ as

$$\begin{aligned} \sigma_{\gamma_P\gamma_P \rightarrow \phi} &= \frac{1}{2s} \overline{|M|^2} (2\pi)^4 \int \frac{d^3\bar{p}}{2(2\pi)^3 E_\phi} \delta^4(p - (k_1 + k_2)) \\ &= \frac{1}{2s} \left(\frac{1}{3}\right)^2 \left(\frac{2\omega_0^2}{\langle\phi\rangle}\right)^2 \left\{ 2 + \left(1 - \frac{s}{2\omega_0^2}\right)^2 \right\} 2\pi \delta(s - m_\phi^2) \end{aligned} \quad (22)$$

The volume emissivity of a supernova with a temperature T through this process is obtained by thermal-averaging over the Bose-Einstein distribution. Hence, the energy loss rate ($\dot{\epsilon}_{\gamma_P} = \frac{1}{\rho_{SN}} \dot{Q}_{\gamma_P}$) due to plasmon plasmon annihilation is given by (similar to that of the energy loss rate via $\gamma\gamma \rightarrow \nu\bar{\nu}$. [25])

$$\begin{aligned} \dot{\epsilon}_{\gamma_P\gamma_P \rightarrow \phi} &= \frac{1}{\rho_{SN}} \langle n_{\gamma_P} n_{\gamma_P} \sigma_{\gamma_P\gamma_P \rightarrow \phi} V_{rel} E_{c.m} \rangle \\ &= \frac{1}{\rho_{SN}} \frac{1}{\pi^4} \int_{\omega_0}^{\infty} d\omega_1 \frac{\omega_1(\omega_1^2 - \omega_0^2)^{1/2}}{e^{\omega_1/T} - 1} \int_{\omega_0}^{\infty} d\omega_2 \frac{\omega_2(\omega_2^2 - \omega_0^2)^{1/2}}{e^{\omega_2/T} - 1} \frac{S(\omega_1 + \omega_2)}{2\omega_1\omega_2} \sigma_{\gamma_P\gamma_P \rightarrow \phi} \end{aligned} \quad (23)$$

where $\sigma_{\gamma_P\gamma_P \rightarrow \phi}$ is given in Eq. 22. Note that $N_{\gamma_P} = \frac{1}{\pi^2} \int_{\omega_0}^{\infty} d\omega \frac{\omega(\omega^2 - \omega_0^2)^{1/2}}{e^{\omega/T} - 1}$ is the number density of thermal photons or plasmons. In the present case, we treat the plasmon to be transverse (with the dispersion relation given by $\omega^2 = \omega_0^2 + |\mathbf{k}|^2$), since the contribution coming from the longitudinal plasmon is typically smaller [26, 27]. In above ω_0 corresponds to plasma frequency in the supernovae core.

Finally introducing the dimensionless variables $x_i = \omega_i/T$ ($i = 0, 1, 2$) and performing the x_2 integration, we find the reaction rate as

$$\dot{\epsilon}_{\gamma_P} = \frac{T^7 x_0^4}{9\pi^3 \langle\phi\rangle^2 \rho_{SN}} \mathcal{F} \int_{x_0}^{\infty} dx_1 \frac{(x_1^2 - x_0^2)^{1/2}}{e^{x_1} - 1} \frac{[(\frac{m_\phi}{T} - x_1)^2 - x_0^2]^{1/2}}{e^{\frac{m_\phi}{T} - x_1} - 1}, \quad (24)$$

where $\mathcal{F} = \left[3 - \left(\frac{m_\phi}{x_0 T}\right)^2 + \frac{1}{4} \left(\frac{m_\phi}{x_0 T}\right)^4 \right]$. In above we have used $\delta[s - m_\phi^2] = \frac{1}{T^2} \delta\left[(x_1 + x_2)^2 - \frac{m_\phi^2}{T^2}\right]$ (while doing the x_2 integration) where $s = (\omega_1 + \omega_2)^2 = T^2(x_1 + x_2)^2$. Also $\omega_0 = Tx_0$ where ω_0 is taken to be equal to m_A (the transverse plasmon mass).

V. NEUTRINO PAIR PRODUCTION IN ELECTRON-POSITRON COLLISION

The neutrino pair production via the s -channel exchange of a light stabilized radion ϕ produced in electron(e^-)-positron(e^+) collision may play a crucial role in the SN1987A cooling. The process is

$$e^-(p_1) + e^+(p_2) \xrightarrow{\phi} \nu_l(p_3) + \bar{\nu}_l(p_4). \quad (25)$$

The radion-fermion-fermion $f(p_1) - \bar{f}(p_2) - \phi(k)$ is given by [3]

$$-\frac{3i}{2\langle\phi\rangle} \left[\not{p}_1 - \not{p}_2 - \frac{8}{3}m_f \right] \quad (26)$$

where p_1, p_2 are the incoming momenta. In the c.o.m frame, the momentum vectors for this reactions are

$$p_1^\mu = (E_1, 0, 0, |\vec{p}|), \quad p_2^\mu = (E_2, 0, 0, -|\vec{p}|), \quad (27)$$

$$p_3^\mu = (E_3, 0, 0, |\vec{p}'|), \quad p_4^\mu = (E_4, 0, 0, -|\vec{p}'|). \quad (28)$$

The total squared amplitude (averaged over the initial spin states and summed over final spin states) for the process is

$$\overline{|M|^2} = \frac{1}{4} \sum_{spins} |M|^2 = \left(\frac{1}{2}\right)^2 \left(\frac{4m_e^2 m_{\nu_l}^2}{\langle\phi\rangle^4}\right) \frac{(s - 4m_e^2)(s - 4m_{\nu_l}^2)}{[(s - m_\phi^2)^2 + m_\phi^2 \Gamma_\phi^2]} \quad (29)$$

Finally, the volume energy-loss rate per unit mass of a supernovae at a temperature T is given by

$$\dot{\epsilon} = \frac{1}{\rho_{SN}} \prod_{i=1}^4 \int \frac{d^3\mathbf{p}_i}{2E_i(2\pi)^3} f_1 f_2 (1 - f_3)(1 - f_4) (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \overline{|M|^2} \quad (30)$$

which after some rearranging can be written as

$$\begin{aligned} \dot{\epsilon} = & \frac{T^7}{64\pi^5 \rho_{SN}} * \iint \frac{x_1 x_2}{[1 + \exp(x_1 - \frac{\mu_{e^-}}{T})] [1 + \exp(x_2 + \frac{\mu_{e^-}}{T})]} \\ & * \frac{\exp(x_1 + x_2) dx_1 dx_2}{[1 + \exp(\frac{x_1 + x_2}{2} + \frac{\mu_{\nu_l}}{T})] [1 + \exp(\frac{x_1 + x_2}{2} - \frac{\mu_{\nu_l}}{T})]} \overline{|M|^2} \end{aligned} \quad (31)$$

where

$$\begin{aligned} \overline{|M|^2} = & \frac{m_{\nu_l}^2 m_e^2}{T^3 \langle\phi\rangle^4 [(T^2(x_1 + x_2)^2 - m_\phi^2)^2 + m_\phi^2 \Gamma_\phi^2]} \\ & \times [T^4(x_1 + x_2)^4 - 4T^2(x_1 + x_2)^2(m_e^2 + m_{\nu_l}^2) + 16m_e^2 m_{\nu_l}^2] \end{aligned}$$

Here $s = (E_1 + E_2)^2 = T^2(x_1 + x_2)^2$ and ω_0 is defined above.

VI. NEUTRINO PAIR PRODUCTION IN PLASMON-PLASMON COLLISION

The third process that we are interested to look at is the neutrino pair production in plasmon-plasmon collision via the s -channel exchange of a light stabilized radion ϕ *i.e.*

$$\gamma_P(k_1) + \gamma_P(k_2) \xrightarrow{\phi} \nu_l(p_1) + \bar{\nu}_l(p_2). \quad (32)$$

The total squared amplitude (averaged over the initial polarization states and summed over the final spin states) is given by

$$\overline{|M|^2} = \left(\frac{1}{3}\right)^2 \sum_{spins} |M|^2 = \frac{16m_A^2 m_{\nu_l}^2}{9\langle\phi\rangle^4} \frac{(s - 4m_{\nu_l}^2)}{[(s - m_\phi^2)^2 + m_\phi^2 \Gamma_\phi^2]} \left[1 + \frac{1}{2} \left(1 - \frac{s}{2m_A^2}\right)^2\right] \quad (33)$$

With this the energy loss rate (volume emissivity) Q due to this process can be written as

$$Q = \prod_{i=1}^2 \int \frac{2^2 d^3 \mathbf{k}_i}{2E_i (2\pi)^3} \prod_{i=3}^4 \int \frac{d^3 \mathbf{p}_i}{2E_i (2\pi)^3} f_1 f_2 (1 - f_3)(1 - f_4) \\ \times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \frac{1}{4} \sum_{spins} |M|^2 \quad (34)$$

where $f_{1,2}$, the occupation numbers for the initial state plasmons and $(1 - f_{3,4})$, the Pauli blocking factors of the final state neutrino and anti-neutrinos.

Finally, the energy loss rate(volume emissivity) per unit mass at temp. T is given by

$$\dot{\epsilon} = \frac{T^7}{16\pi^5 \rho_{SN}} * \iint dx_1 dx_2 \frac{x_1 x_2}{[\exp(x_1) - 1] [1 + \exp(x_2 + \frac{\mu_{e^-}}{T})]} \\ * \frac{\exp(x_1 + x_2) dx_1 dx_2}{[1 + \exp(\frac{x_1 + x_2}{2} + \frac{\mu_{\nu_l}}{T})] [1 + \exp(\frac{x_1 + x_2}{2} - \frac{\mu_{\nu_l}}{T})]} \overline{|M|^2} \quad (35)$$

where

$$\overline{|M|^2} = \frac{16m_{\nu_l}^2 m_A^4}{9T^3 \langle\phi\rangle^4} \frac{[T^2(x_1 + x_2)^2 - 4m_{\nu_l}^2]}{[(T^2(x_1 + x_2)^2 - m_\phi^2)^2 + m_\phi^2 \Gamma_\phi^2]} \\ \times \left[1 + \frac{1}{2} \left(1 - \frac{T^2(x_1 + x_2)^2}{2m_A^2}\right)^2\right]$$

where $s = (E_1 + E_2)^2 = T^2(x_1 + x_2)^2$.

VII. NUMERICAL ANALYSIS

As discussed above the processes of our interest that may contribute in the SN1987A cooling are radion mediated three processes: (i) $\gamma_P + \gamma_P \rightarrow \phi$, (ii) $e^- + e^+ \xrightarrow{\phi} \nu_x + \bar{\nu}_x$ and

(iii) $\gamma_P + \gamma_P \xrightarrow{\phi} \nu_x + \bar{\nu}_x$ (with $x = \mu, \tau$). If the SN1987A cooling is due to a class of radion mediated processes the emissivity rate for those channels must be $\dot{\epsilon} \leq 10^{19} \text{ erg g}^{-1} \text{ s}^{-1}$ [5], which can be converted to a lower bound on radion vev $\langle\phi\rangle$ as discussed below.

A. Bound on $\langle\phi\rangle$ from the $\gamma_P + \gamma_P \rightarrow \phi$ decay

In Fig. 2 we have shown the energy-loss rate $\dot{\epsilon}(\gamma_P + \gamma_P \rightarrow \phi)$ as a function of the radion $\langle\phi\rangle$ corresponding to an ultra-light stabilized radion of mass $m_\phi = 1.25, 1.5, 1.75, 2.0$ and 2.5 GeV. The upper horizontal line corresponds to the upper bound of the supernovae energy

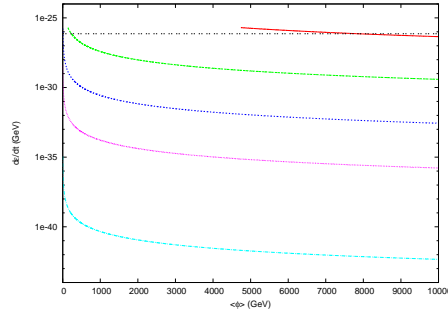


FIG. 2: The supernovae energy-loss rate $d\epsilon/dt \text{ GeV}^{-1}$ due to radion emission produced in plasmon-plasmon annihilation is shown as a function of $\langle\phi\rangle$ (GeV). While going from the topmost curve to the lowermost curve, m_ϕ increases as 1.25, 1.5, 1.75, 2.0 and 2.5 GeV. The upper horizontal line corresponds to the upper bound of the supernovae energy loss rate i.e. $\dot{\epsilon} \leq 7.288 \times 10^{-27} \text{ GeV}$.

loss rate $\dot{\epsilon} = 7.288 \times 10^{-27} \text{ GeV}$ for any new physics channel. From the topmost curve to the lowermost curve the radion mass m_ϕ increases as 1.25, 1.5, 1.75, 2.0 and 2.5 GeV. We find $\langle\phi\rangle > 7852 \text{ GeV}$ for $m_\phi = 1.25 \text{ GeV}$ and $\langle\phi\rangle > 230 \text{ GeV}$ for $m_\phi = 1.5 \text{ GeV}$. No bound follows for $m_\phi = 1.75, 2.0$ and 2.5 GeV .

B. Bound on $\langle\phi\rangle$ from the $e^- + e^+ \xrightarrow{\phi} \nu_l + \bar{\nu}_l$ scattering

The muon and tau neutrino or anti-neutrino produced in $e^- e^+$ annihilation (due to a s channel exchange of a light stabilized radion) may take away the supernovae energy and hence can explain the deficit of electron type of (anti)neutrino at the detector recorded by the Kamiokande and IMB collaborations. On the left panel of Fig. 3 we have shown the energy loss rate $\dot{\epsilon}(e^- + e^+ \xrightarrow{\phi} \nu_\mu + \bar{\nu}_\mu)$ as a function of $\langle\phi\rangle$, whereas on the right panel we have shown the energy loss rate $\dot{\epsilon}(e^- + e^+ \xrightarrow{\phi} \nu_\tau + \bar{\nu}_\tau)$ as a function of $\langle\phi\rangle$. The horizontal

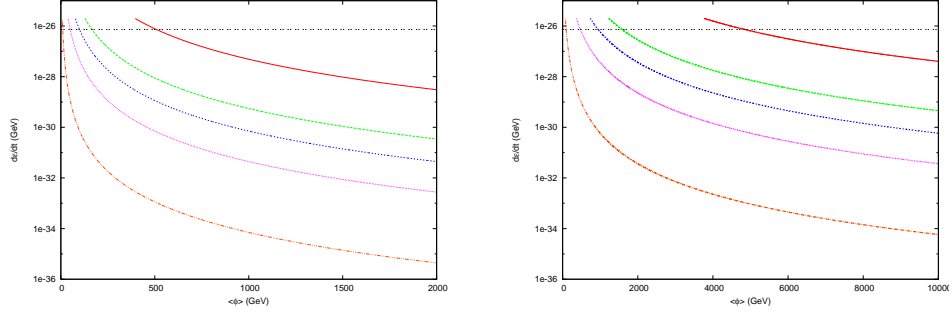


FIG. 3: The energy-loss rate $d\epsilon/dt$ GeV^{-1} is shown as a function of $\langle\phi\rangle$ (GeV). From the topmost to the lowermost curve, m_ϕ increases as 1, 3, 5, 10 and 50 GeV, respectively. The horizontal line corresponds to $\dot{\epsilon} \leq 7.288 \times 10^{-27} \text{GeV}$. The left panel corresponds to the energy loss due to $e^- + e^+ \xrightarrow{\phi} \nu_\mu + \bar{\nu}_\mu$ scattering, whereas the right panel corresponds to the energy loss by $e^- + e^+ \xrightarrow{\phi} \nu_\tau + \bar{\nu}_\tau$ scattering.

line on each panel corresponds to the upper bound $\dot{\epsilon} \leq 7.288 \times 10^{-27} \text{GeV}$. On the left(right) panel, from the topmost to the lowermost curves m_ϕ increases as 1, 3, 5, 10 and 50 GeV.

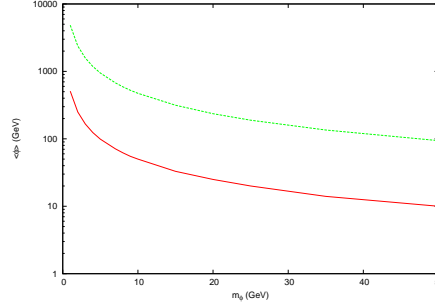


FIG. 4: The lower bound on $\langle\phi\rangle$ is plotted against m_ϕ . The lower and the upper curves follows from the fact that the supernovae cooling stems from the $e^- + e^+ \xrightarrow{\phi} \nu_\mu + \bar{\nu}_\mu$ and $e^- + e^+ \xrightarrow{\phi} \nu_\tau + \bar{\nu}_\tau$ scattering processes, respectively. The region above each curve is allowed.

In Fig. 4, we have shown m_ϕ as a function of $\langle\phi\rangle$ (the lower bound). The bound follows from the fact if any new physics channel contribute to the SN1987A energy loss, it should be $\dot{\epsilon} \leq 7.288 \times 10^{-27} \text{GeV}$. The lower(upper) curve follows from the fact that the SN1987A cooling is caused by the $e^- + e^+ \xrightarrow{\phi} \nu_\mu(\nu_\tau) + \bar{\nu}_\mu(\bar{\nu}_\tau)$ radion mediated processes. The region above each curve is allowed.

Below in Table 2 we have shown the lower bound on $\langle\phi\rangle$ for different m_ϕ (refer Fig. 4). From Table 2 we see that for a given channel, the lower bound on $\langle\phi\rangle$ decreases with the increase in m_ϕ . Also for a particular m_ϕ the lower bound on $\langle\phi\rangle$ (third column) which follows from $\nu_\tau - \bar{\nu}_\tau$ production is much higher than that(second column) follows from $\nu_\mu - \bar{\nu}_\mu$

production. Note that the bound on $\langle\phi\rangle$ (second column) is much weaker for a radion of mass of few tens of a GeV.

Table 2

m_ϕ (GeV)	$\langle\phi\rangle$ (GeV) (lower curve)	$\langle\phi\rangle$ (GeV) (upper curve)
1	509	4846
3	166	1580
5	99	946
10	50	473
15	33	315
20	25	236
25	20	189

Table 2: The lower bound on $\langle\phi\rangle$ (follows from Fig. 4) corresponding to different m_ϕ is shown. The second(third) column follows from the energy loss rate $\dot{\epsilon}(e^- + e^+ \xrightarrow{\phi} \nu_{\mu(\tau)} + \bar{\nu}_{\mu(\tau)}) \leq 7.288 \times 10^{-27} \text{GeV}$.

C. Bound on $\langle\phi\rangle$ from the $\gamma_P + \gamma_P \xrightarrow{\phi} \nu_l + \bar{\nu}_l$ scattering

The Supernovae may cool off by emitting muon and tau neutrino and anti- neutrino produced in plasmon-plasmon annihilation inside it's core. The relevant processes are $\gamma_P + \gamma_P \xrightarrow{\phi} \nu_x + \bar{\nu}_x$ (with $x = \mu, \tau$). In Fig. 5, we have shown the energy-loss rate as a function of $\langle\phi\rangle$ corresponding to different m_ϕ values. With the lowering of radion mass,

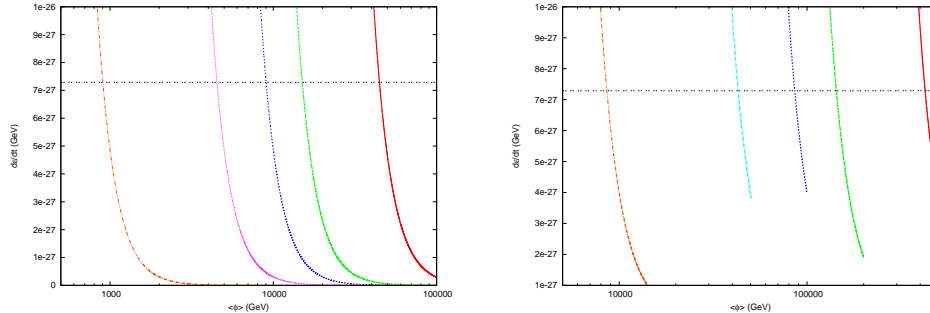


FIG. 5: The energy-loss rate $d\varepsilon/dt$ GeV^{-1} is shown as a function of $\langle\phi\rangle$ (GeV). From the rightmost to the leftmost curve, m_ϕ increases as 1, 3, 5, 10 and 50 GeV, respectively. The horizontal line corresponds to the upper bound on the energy loss rate i.e. $\dot{\epsilon} \leq 7.288 \times 10^{-27} \text{GeV}$. The left(right) panel corresponds to the energy loss by $\gamma_P + \gamma_P \xrightarrow{\phi} \nu_\mu + \bar{\nu}_\mu$ ($\gamma_P + \gamma_P \xrightarrow{\phi} \nu_\tau + \bar{\nu}_\tau$) processes.

the lower bound on $\langle\phi\rangle$ decreases. In Fig. 6, we have plotted $\langle\phi\rangle$ (the lower bound) as a function of radion mass m_ϕ . The lower and the upper curves correspond to the SN1987A

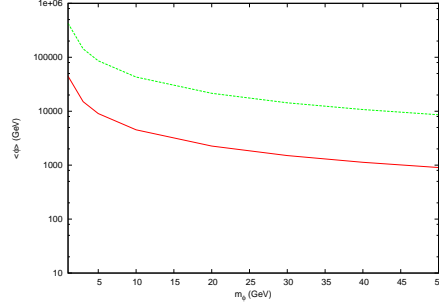


FIG. 6: The lower bound on $\langle\phi\rangle$ is plotted against m_ϕ . The lower bound follows from the fact for any new physics channel contributing to the SN1987A energy loss, the loss rate $\dot{\epsilon} \leq 7.288 \times 10^{-27} \text{ GeV}$. The lower(upper) curve follows from the fact that the processes $\gamma_P + \gamma_P \xrightarrow{\phi} \nu_\mu(\nu_\tau) + \bar{\nu}_\mu(\bar{\nu}_\tau)$ contribute to the SN1987A cooling. The region above each curve is allowed.

cooling due to $\gamma_P + \gamma_P \xrightarrow{\phi} \nu_\mu + \bar{\nu}_\mu$ and $\gamma_P + \gamma_P \xrightarrow{\phi} \nu_\tau + \bar{\nu}_\tau$ processes, respectively. In Table 3 we have shown the lower bound on $\langle\phi\rangle$ for different m_ϕ values. For an ultra-light radion of mass 1 GeV, we find the lower bound on $\langle\phi\rangle$ as 44.6 TeV and ~ 425 TeV, respectively whereas for a radion of mass about 50 GeV, we find the lower bound as 0.9 TeV and 8.6 TeV, respectively.

Table 3

m_ϕ (GeV)	$\langle\phi\rangle$ (GeV) (left panel)	$\langle\phi\rangle$ (GeV) (right panel)
1	44648	424951
3	15042	143147
5	9033	85958
10	4518	42993
20	2259	21498
30	1506	14332
50	904	8600

Table 3: The lower bound on $\langle\phi\rangle$ as a function of m_ϕ corresponding to the SN1987A energy loss rate $\dot{\epsilon} \leq 7.288 \times 10^{-27} \text{ GeV}$. The second(third) column of the lower bound on $\langle\phi\rangle$ follows from the energy loss via the channel $\gamma_P + \gamma_P \xrightarrow{\phi} \nu_{\mu(\tau)} + \bar{\nu}_{\mu(\tau)}$.

VIII. CONCLUSION

In this work we have investigated the impact of a light stabilized radion in supernovae cooling. The radion produced inside the core of a supernova due to electron-positron and plasmon-plasmon collisions, can take away much of the energy released in supernovae explosion. The primary processes of our concern are (i) $\gamma_P + \gamma_P \rightarrow \phi$, (ii) $e^+ + e^- \xrightarrow{\phi} \nu_{\mu(\tau)} + \bar{\nu}_{\mu(\tau)}$ and (iii) $\gamma_P + \gamma_P \xrightarrow{\phi} \nu_{\mu(\tau)} + \bar{\nu}_{\mu(\tau)}$. Assuming that the energy loss rate due to each of the above three channels $\dot{\epsilon} \leq 7.288 \times 10^{-27} \text{ GeV}$, we obtain the following lower bound on the radion vev $\langle \phi \rangle$ due to a light stabilized radion. For the process $\gamma_P + \gamma_P \rightarrow \phi$: we find $\langle \phi \rangle \geq 7.85 \text{ TeV}$ for $m_\phi = 1.25 \text{ GeV}$ and $\langle \phi \rangle \geq 2.3 \text{ GeV}$ for $m_\phi = 1.5 \text{ GeV}$. For $e^+ + e^- \xrightarrow{\phi} \nu_x + \bar{\nu}_x$: with $x = \tau$ we find $\langle \phi \rangle \geq 0.95 \text{ TeV}$ and $\geq 0.24 \text{ TeV}$ corresponding to $m_\phi = 5 \text{ GeV}$ and 20 GeV , respectively. No reasonable bound on $\langle \phi \rangle$ follows with $\nu_\mu, \bar{\nu}_\mu$ as final state particles. Finally the process $\gamma_P + \gamma_P \xrightarrow{\phi} \nu_x + \bar{\nu}_x$: for $x = \mu$, we find $\langle \phi \rangle \geq 9.0 \text{ TeV}$ and $\geq 2.4 \text{ TeV}$ for $m_\phi = 5 \text{ GeV}$ and 20 GeV , respectively. For $x = \tau$, we find $\langle \phi \rangle \geq 86 \text{ TeV}$ and $\geq 21.5 \text{ TeV}$ corresponding to the same m_ϕ values.

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